

USING THE PAIRED COMPARISON METHOD IN IMPROVING SOCCER RANKINGS: THE CASE OF THE ROMANIAN FIRST DIVISION

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ABSTRACT: The most common way to establish the final ranking in the football leagues across the world is to have all participating teams playing each other, home and away, in a double round-robin system. The team which obtains the highest number of points is declared champion while a number of teams that have obtained the lowest number of points are relegated. Various criteria are considered to differentiate the equal teams, such as superior overall goal difference, higher number of goals scored during the competition, results in the direct games, and so on. Complementary ways are considered to improve the final ranking, such as a double number of home-away games, play-offs and play-outs, supplementary games played home or away. The paper advances a new way to determine the final ranking of a football leagues based on the paired comparison approach and presents the results of testing this approach at the level of the Romanian First Division.

Keywords: paired comparison method, soccer ranking, sports marketing

JEL Codes: M31, L8

Introduction

The most common (and, in theory, the fairest) way to establish the final ranking of a football league is to have all participating teams playing each other, home and away, in order to capitalize the advantage of the own stadium, in a double round-robin system. Champion is declared the team which, at the end of the tournament, obtains the highest number of points based on a specific allocation (usually three points for win, one for draw and zero for loss). At least one and up to several number of teams that have obtained the lowest number of points are relegated in the next inferior league.

Various criteria are considered to differentiate the equal teams in terms of the points earned at the end of the competition and to determine the final ranking, such as: superior goal difference from all the matches played, higher number of goals scored in all the matches played, results in the direct games, and so on.

Various complementary ways are considered to generate an improved final ranking or to increase the competitiveness of the competition, such as: a double number of home-away games (as in Austrian Bundesliga or in the Swiss Super League), a third game played home or away based on a certain rule (such as in Danish Superligaen), a number of supplementary games with the teams in the first or the second half of the hierarchy established after a series of three games played (as in the Scottish Premier League) or playoffs and play-outs disputed in a second phase of the competition (planned to be implemented in the Romanian First Division starting with the season 2015-2016).

At the end of the season, the final ranking indicates the champion, teams qualified in the

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international club competitions (Champions League and/or Europa League), respectively, in the case of lower leagues, the teams directly promoted or playing-off for promotion in the superior leagues, and teams relegated (directly or playing-out for avoiding relegation) in the inferior leagues. Somewhere in the middle area of the ranking is the group, which may vary in size, of around a half or even more of the participating teams without any other performance but maintaining their positions in the league.

So far, the round robin tournament system has appeared to work well enough although questions on the merits of the title winner, teams qualified in European competitions or relegated, raised mainly by the fans and media, have been often heard at the end of almost each season. This traditional format of the football leagues seemed to provide a balanced background for playing in the championship: the amount of points acquired at the end of the season rewarded each team's effort and served for building a reasonably fair final ranking.

Can the current ranking system of the football leagues be improved by using a paired comparison approach? The traditional format of the football leagues, as regularly scheduled competitions played by "n" teams in order to determine a ranking, can be simply assimilated to a number of "n" stimuli that are to be assessed in a comparative manner to identify the best, the poorest and their overall ranking. That is why a pair-wise comparison approach could be easily imported and integrated as a mechanism to establish the final ranking of a football league without being necessary to change any aspects regarding the organization of the competition.

Literature review

The theory of tournament rankings represents a topic with a respectable and consistent history, inside which a particular area – round robin tournaments – has received a special attention, illustrated by the studies of Moon (1968), Daniels (1969), Rubenstein (1980), Goddard (1983), Stob (1985) and Slutzki and Volij (2005). Peterson (1993) and Stern (1995) have attempted to answer who's really No.1 and how the statistics can influence the rankings of the college football competitions or how it can be decided in this respect. More recently, Brozos-Vázquez et al. (2008) have proposed a new system of tournament ranking by means of rating functions based on the recursive performance, while Sitarz (2013) has addressed the problem of building the Olympic ranking using a system of points given for medals based on the in-center of a convex cone, with application in scoring systems in sport.

While the most part of the research conducted to study and propose improvements of the tournament rankings had a strong quantitative content, there were several attempts to consider also the qualitative dimension of the ranking. Thus, Lebovic and Sigelman (2003) have identified the inertia and constraint, rank elevators, and the passage of time as predictors for positioning in the league rankings and also observed that lower-ranked teams move up faster after a victory than do higher-ranked teams, but moving up in the rankings after a victory is an incremental process — much more incremental than moving down after a loss. Analyzing comparatively the Spanish and Italian leagues, Boscá et al. (2009) have observed using mathematical optimization methods that getting a better position in the final ranking depends more on improving offensive efficiency (in Primera Division), respectively defensive efficiency (in the Serie A).

From another angle, Da Silva, Matsushita and Silveira (2013) have seen sports as stylized combat, which may follow power laws similar to those of wars, individual clashes, and acts of terrorism. Their research conducted at the level of the top fifteen European football leagues has illustrated existence of the hidden power laws based on the close relationship between rank and points won by the participating clubs.

The pair-wise comparison approach has been taken into consideration by Dahl (2012), which has introduced a ranking methodology applicable in sports like tennis based on linear algebra

and involving the computing of a score for each player by solving a certain linear system of equations leaving from a set of matches and weights representing their importance. Aiming to answer who is the greatest tennis player, Baker and McHale (2014) took into consideration the models for paired comparisons in the case when many competitors play in a series of head-to-head competitions and extended them to allow for time-varying competitor strengths.

Coming closer to the football, Hvattum and Arntzen (2010) have examined the value of assigning ratings to teams based on their past performance in order to predict match results in association football and proposed a score system, inspired by the ELO rating system, in which a win gives a score of 1, a draw a score of 0.5, and a loss a score of 0 and suggest a possible improvement by updating the scores with a coefficient k depending on the goal difference. Glickman and Jensen (2005) have already drew attention over the design of the paired comparison observing that when designing a tournament or league schedule, teams could be ranked through the results of previous competition or expert judgment and proposed a method for designing paired comparison experiments, particular to tournament scheduling.

The previous research attempts have approached the problem of tournament rankings from a statistical and mathematical perspective aiming mainly: (1) to provide quantitative explanations on the final rankings in different sports (particularly in tennis and chess), (2) to improve the statistical accuracy of the final rankings of the different competitions, and (3) to identify elements to be considered in predicting the match results and/or the final ranking of different sports competitions.

Methodological notes

The research aimed to explore a new way to determine the final ranking of the football leagues based on the paired comparison method, respectively the pair-wise comparison approach. The *pair-wise comparison* generally refers to the process in which different *paired entities* are assessed to establish which of each entity is preferred or has a greater amount of some quantitative and/or qualitative property.

Generally, given two alternatives “A” and “B”, there are the following pair-wise comparisons: (1) “A” is better than “B”, (2) “B” is better than “A” and (3) “A” is equal to “B”. Considering these alternatives in the context of a football league where “ n ” teams play each other during the season, each participating team will have to dispute “ $n-1$ ” *pair-games*. Under these circumstances, winning the competition and becoming the leagues’ champions requires to be the best team in as many as pair-games, *ideally in all*. The league’s final ranking will be established considering the descending order of the pair-game winnings, which will be rewarded with *pair-wise points* (of whose maximum amount will be equal to $n-1$).

The main objectives of the research, considering the Romanian First Division as case study, were: (O1) to build a new league ranking based on a pair-wise comparison approach; (O2) to test the statistical significance of the differences between the traditional and the pair-wise comparison rankings; and (O3) to identify and assess the differences between the traditional and the pair-wise comparison rankings outputs (champions, teams qualified in the European competitions, respectively relegated teams).

The corresponding research hypotheses stated that: (H1) the pair-wise comparison ranking system leads to a different hierarchy by comparison to the traditional ranking system; (H2) there is a significant difference between the hierarchies determined using the traditional and pair-wise comparison ranking systems, and (H3) there are significant differences in terms of the competition’s output taking into consideration the traditional, respectively pair-wise comparison ranking system.

Data provided by The Rec.Sport.Soccer Statistics Foundation (<http://rssf.com/>), cross-checked with those provided by the official website of the Romanian First Division (www.lpf.ro) have been collected and processed in order to build the cross table of results of the analyzed edition

(as a matrix presented in the Figure 1, where H_{in} is the score of the match between teams “i” and “n” on the “i” team home stadium). Next, a comparative cross table has been built taking into consideration the scores registered in each pair-game and the pair-wise comparison ranking system (as a Matrix presented in the Figure 2, where PwP_{in} is the amount of the pair-wise points obtained by the “i” team after the pair-games played with the “n” team”, respectively 1, 0.5 or zero).

	Team 1	Team 2	Team i	Team n
Team 1		H_{12}	H_{1i}	H_{1n}
Team 2	H_{21}		H_{2i}	H_{2n}
Team i	H_{i1}	H_{i2}		H_{in}
Team n	H_{n1}	H_{n2}	H_{ni}	

Figure No. 1 The cross table of results of the football competition

Each of the n-1 pair-games played by the league teams will have a “winner” or a “loser” established considering the *aggregate score* (and the corresponding goal difference) and the *away goal rule* stating that the team that has scored more goals “away from home” will win if scores are otherwise tied. The winner of the pair-game will receive one pair-wise point, while the loser will receive zero pair-wise points. In the situations of perfectly symmetrical results (the same scores registered in the home and away matches), each team will receive an equal number of pair-wise points, respectively 0.5.

	Team 1	Team 2	Team i	Team n
Team 1		PwP_{12}	PwP_{1i}	PwP_{1n}
Team 2	PwP_{21}		PwP_{2i}	PwP_{2n}
Team i	PwP_{i1}	PwP_{i2}		PwP_{in}
Team n	PwP_{n1}	PwP_{n2}	PwP_{ni}	

Figure No. 2 Matrix of the paired comparison cross table

The league’s intermediary and final rankings have been established at the end of each pair-games’ round (the number of the pair-games rounds being equal to n-1) considering the descending order of the pair-wise points obtained by each participating team.

The pair-wise comparative ranking of the Romanian First Division for the 2013-2014 season has been “reconstructed” under these conditions based on the overall aggregate scores registered in all 153 pair-games played during the competition and the corresponding amount of pair-wise points earned by each team during the competition. The points earned in the traditional three-points-for-win system, as well as the goal difference from all the matches played, have also been considered as the second and third differentiators of the equal teams, the first being the status of the teams (“winner” or “loser”) in the pair-game directly disputed or the descending order of the pair-wise points obtained by the three or more equal teams.

Main findings of the research

There are two essential conclusions that can be drawn making the comparison between the traditional and pair-wise comparative rankings of the 2013-2014 Romanian First Division edition (as they presented in the Figure No. 3): (1) the final ranking of the competition would have been

different and (2) the competition's „verdicts” would also have been different!

15 out of the 18 participating teams (83.3 %) would have get a different position in the final ranking of the competition. Although the most of these teams would have ended the competition just one rank up or down, there were teams of whose positions would have been considerably different: Săgeata Năvodari would have finished six ranks up and avoided the relegation, while Viitorul Constanța would have ended five ranks lower, exactly in the Săgeata's place and relegated!

Round 34							Pair-round 17				
Teams	P	W	D	L	GD	Pts	Verdicts	Teams	PwP	Pts	GD
1 Steaua București	34	22	11	1	71-20	77	Champions	1 Petrolul Ploiești	15	68	53-20
2 Astra Giurgiu	34	22	6	6	70-28	72	Europa League	2 Steaua București	13.5	77	71-20
3 Petrolul Ploiești	34	18	14	2	53-20	68	Europa League	3 Astra Giurgiu	13	72	70-28
4 Dinamo București	34	17	8	9	52-34	59		4 Dinamo București	12	59	52-34
5 CFR Cluj	34	13	12	9	44-35	51		5 SC Vaslui	10.5	51	38-32
6 SC Vaslui	34	15	6	13	38-32	51		6 Pandurii Târgu Jiu	10	50	59-39
7 Pandurii Târgu Jiu	34	14	8	12	59-39	50		7 CFR Cluj	9	51	44-35
8 ASC Botoșani	34	12	7	15	36-52	46		8 Oțelul Galați	8.5	41	43-52
9 Ceahlăul Piatra Neamț	34	10	11	13	27-31	41		9 Universitatea Cluj	8	40	29-46
10 Oțelul Galați	34	12	5	17	43-52	41		10 Ceahlăul Piatra Neamț	7.5	41	27-31
11 Universitatea Cluj	34	11	7	16	29-46	40		11 Săgeata Năvodari	7.5	38	32-54
12 Viitorul Constanța	34	10	10	14	29-50	40		12 Concordia Chiajna	7	39	34-47
13 Gaz Metan Mediaș	34	10	9	15	32-38	39		13 ASC Botoșani	7	43	36-52
14 Concordia Chiajna	34	10	9	15	34-47	39		14 FC Brașov	7	38	32-40
15 FC Brașov	34	9	11	14	32-40	38	Relegated	15 ACS Poli Timișoara	6.5	38	26-42
16 ACS Poli Timișoara	34	10	8	16	26-42	38	Relegated	16 Gaz Metan Mediaș	5.5	39	32-38
17 Săgeata Năvodari	34	10	8	16	32-54	38	Relegated	17 Viitorul Constanța	5	40	29-50
18 Corona Brașov	34	2	8	24	20-69	14	Relegated	18 Corona Brașov	0.5	14	20-69

Notes: P - matches played; W - matches won; D - draws; L - defeats; GD - goal difference; Pts - total points earned in traditional system;

PwP - pair-wise points (earned under a comparative approach).

Figure No 3. The traditional versus pair-wise comparison ranking of the Romanian First Division for the season 2013-2014

As about the league „verdicts”, the Champions of Romania would have been Petrolul Ploiești, the runner-ups, respectively the teams qualified in the European club competitions (taking into consideration the decisions of the Licensing Commission), would have been, besides Astra Giurgiu and Steaua București, Pandurii Târgu Jiu (instead of CFR Cluj), while the relegated teams would have been SC Vaslui, Gaz Metan Mediaș, Viitorul Constanța and Corona Brașov!

The away goal rule would have decided the winners of only 17 out of the 153 pair-games played (11.1 %). In a hierarchy of the benefits and losses generated by the enforcement of this rule (expressed as a net difference between the pair-wise points won and lost), the most favored teams would have been Universitatea Cluj (+2), Dinamo București (+1), Oțelul Galați (+1), Săgeata Năvodari (+1) and Concordia Chiajna (+1). Petrolul Ploiești (0), Pandurii Târgu Jiu (0), Ceahlăul Piatra Neamț (0), ACS Poli Timișoara (0) and Gaz Metan Mediaș (0) would have balanced benefits and losses, while the list of its “victims” would have included FC Brașov (-1), CFR Cluj (-2) and Steaua București (-3). Steaua was the most illustrious victim losing three pair-wise points without gaining any! Almost a third of the participating teams would not have been affected by the rule – Astra Giurgiu, SC Vaslui, FC Botoșani, Viitorul Constanța and Corona Brașov.

Only 7 out of the 153 pair-games played (4.6 %) have ended as a draw and would have involved the splitting of the pair-wise points. ACS Poli was three times in this situation, U. Cluj and Concordia twice, while Ceahlăul, Corona, Vaslui, Steaua, Gaz Metan, Oțelul and Săgeata just once. The most spectacular perfectly symmetric score was Concordia vs Săgeata (4-0 and 0-4), while the most disappointing was, from all the perspectives, Corona vs ACS Poli (0-0 and 0-0).

The battle for the championship was a business in three on the route Ploiești – București – Giurgiu. In the traditional system, the things were clearly ended after the 31st round by Steaua winning against U. Cluj. Still, under the pair-wise comparison approach, the battle would have ended only in the last, 34th, round with the success of Petrolul, the “Yellow Wolves” being simply superior to 15 out of their 17 competitors (including Steaua!), while Steaua and Astra succeeded to be better than only 13 of their league rivals.

Petrolul has lost just two pair-wise points, vs Ceahlăul (1-1 and 0-0), respectively Dinamo (2-2 and 1-1), Steaua splitted the pair-wise point with Vaslui (0-1 and 1-0), but lost the points vs Pandurii (2-2 and 1-1), Petrolul (1-1 and 0-0) and Oțelul (2-2 and 1-1), while Astra were inferior to Steaua (1-1 and 1-3), Petrolul (1-2 and 1-1), Concordia (1-0 and 1-3), respectively Dinamo (2-1 and 0-2). Petrolul would have deserved the title also because they were superior to both Steaua and Astra, while the team from Giurgiu would have deserved to end on the third position being below both Petrolul and Steaua.

Besides Petrolul, Steaua and Astra, other seven teams have appeared in the first three positions of the comparative ranking: Pandurii (2nd position, after the first pair-games round), Săgeata (3rd position, after the second and third pair-games rounds), ACS Poli (3rd position, after the fourth pair-games round), Ceahlăul (3rd position, after the sixth pair-games round), Botoșani (3rd position, after the seventh pair-games round), Dinamo (3rd position, after the eleventh and twelfth pair-games rounds) and SC Vaslui (after the thirteenth pair-games round). In the end, all these teams would have held positions between the 5th and 15th places in the league’s ranking, with Vaslui qualifying for the Europa League (without having financial difficulties) and ACS Poli relegating in the Second Division.

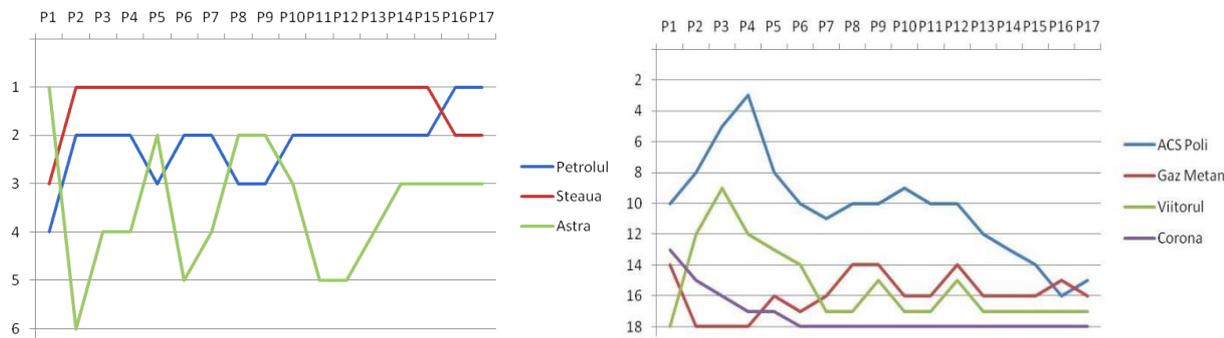


Figure No. 4 The evolution of the first three, respectively the last four teams in the pair-wise comparative ranking during the seventeen pair-games rounds

ACS Poli Timișoara, Gaz Metan Mediaș, Viitorul Constanța and Corona Brașov would have got the lowest positions in the league’s ranking built considering the pair-wise comparison approach and, without any other administrative corrections, relegated in the Second Division.

One of the poorest teams that have ever played in the Romanian First Division, Corona entered the relegation area right after the second pair-games round “succeeding” to be inferior to 16 out of 17 of their league rivals, with the exception represented by the ACS Poli. The team from Brașov fully deserved the relegation, without any doubts, being forced to withdraw even from the Second Division after the local authorities decided to disband the soccer section of the club.

The team managed by Gheorghe Hagi, Viitorul („the Future”), could become “(the) Past” if relegated at the end of the season. The team from Constanța, temporarily relocated at Chiajna (Ilfov) and in full process of re-relocation at Voluntari (Ilfov), was superior only to Concordia (1-1 and 2-1), Dinamo (0-0 and 2-1), Vaslui (2-0 and 1-0), Corona (1-0 and 4-0) and Ceahlăul (1-0 and

0-0). Although poorer than other two relegated teams – Săgeata and ACS Poli, Viitorul managed to survive in the First Division due also to some controversial points raised with Dinamo or Vaslui.

Crawling through the championship, Gaz Metan would have arrived to play hard for avoiding relegation with FC Braşov, quite in the last round. If, in classical format, the 1-1 draw has been enough satisfactory for both teams, which, this way, remained together in the First Division, in the pair-wise comparative ranking, the team from Mediaş would have had to win in order to get the pair-wise point that would have saved Gaz Metan and sent FC Braşov in the Second Division! Gaz Metan was better than Viitorul (4-0 and 0-0), Corona (1-0 and 2-1), Pandurii (0-1 and 2-1), Ceahlăul (3-1 and 1-0), respectively CFR Cluj (2-0 and 1-3) splitting one pair-wise point with Concordia (1-0 and 0-1).

Although holding, at certain moments, a top position both in the traditional and pair-wise comparative rankings, ACS Poli has ended on a place that sent Timișoara's team in the Second Division. The pair-wise points lost vs Botoşani, FC Braşov, Vaslui, Steaua, Pandurii and Petrolul, interpolated with the splittings with Corona, U. Cluj and Oţelul, have accounted too much by comparison to the pale successes obtained vs CFR Cluj (1-0 and 2-2), Gaz Metan (2-1 and 1-0), Săgeata (1-1 and 2-1), Viitorul (0-0 and 2-1), respectively Concordia (1-0 and 1-2). Still, if the pair-wise comparative ranking would have been in function, ACS Poli would have managed to remain in the First Division benefitting from the administrative relegation of the SC Vaslui.

Besides the occupants of the last four positions, five more teams have actively been involved in the battle for avoiding relegation which made a half of the competitors trembling for their presence in the First Division: Universitatea Cluj (holding the 15th position after the first and sixth pair-games rounds), Ceahlăul (16th position after the first pair-games round), Pandurii (15th after the fifteenth pair-games round), Concordia (15th, 16th and 17th places, after the second, third, fourth, seventh, eighth, ninth, eleventh, twelfth, thirteenth and fourteenth pair-games rounds), respectively FC Braşov (15th, 16th, 17th and 18th places, after the first, second, third, fourth, fifth, sixth, eighth, ninth, tenth and twelfth pair-games rounds).

Maybe the most dissonant case is that of Săgeata Năvodari. Although the team from Dobrogea has relegated in the traditional system, they would have ranked eleventh in the pair-wise comparative hierarchy being better than Oţelul (2-1 and 1-0), U. Cluj (1-0 and 1-1), Botoşani (1-0 and 1-1), FC Braşov (0-0 and 1-1), Gaz Metan (1-0 and 2-1), Viitorul (3-1 and 0-0), respectively Corona (1-0 and 0-0) and perfectly equal to Concordia (4-0 and 0-4).

Conclusions, limits of the research and discussion

A quick analysis of outcomes of the 153 pair-games played in the Romanian First Division in the 2013-2014 season, conducted from the perspective of a pair-wise comparative ranking system, reveals at least the following three conclusions:

- (1) If each and every match played would have mattered – as the pair-wise comparative ranking system would have imposed, the premature relaxation that appeared in some of the games played by Steaua, Astra or Dinamo would have been prevented. Steaua and Astra would have disputed the title or at least the participation in the Europa League until the last round, followed closely by a more motivated Dinamo. The sense each match had would have increased the attractiveness of the entire competition and made (almost?) all 306 matches of interest for spectators and media;
- (2) More, if each and every match played would have mattered, the emergence of some „surprising results”, such as Dinamo vs Viitorul or Vaslui vs Viitorul, as well as of the „mutually beneficial results”, such as Gaz Metan vs FC Braşov, would have been far less probable. The significantly lower predictability of the competition would have made it not only more appealing for the public, but also more objective in setting the final ranking of the

participating teams;

- (3) The increased stake and higher fairness of the competition would attract significantly more spectators at the stadiums: it is more than reasonable to suppose that a match fairly disputed on the ground generates a higher interest, a stronger engagement of the fans willing to be closer to their favorite teams and, as a result, an increased desire to watch it live;
- (4) The increased audiences of the teams and their evolutions would offer each club and the entire league much more marketing opportunities and an overall marketing context significantly improved in terms of the money to be raised from ticketing, advertising, merchandising and selling of the media rights.

In terms of the research objectives and hypotheses, the results obtained – the new league ranking build based on the pair-wise comparative approach and the comparisons between the traditional and comparative rankings allow to conclude that:

- (1) the pair-wise comparison ranking system generated a different hierarchy by comparison to the traditional ranking system: with 15 out of the 18 participating teams (83.3 %) holding a different position in the final ranking of the competition, and also with changes in terms of the ranks held of one or two places, but also six places up or five places down, there is a significantly different hierarchy built using the comparative approach;
- (2) the differences between the hierarchies determined using the traditional and pair-wise comparative ranking systems did not prove statistically significant (computed value of $\chi^2=5.24$, theoretical value of $\chi^2=27.59$, for $\alpha=0.05$ and 17 degrees of freedom). The pair-wise comparative ranking system generates a different hierarchy only in terms of the place exchanging, but not of the statistical significance;
- (3) finally, there are significant differences in terms of the competition's output taking into consideration the traditional, respectively pair-wise comparison ranking system. With different champions – Petrolul Ploiești, the runner-ups – Steaua București, teams qualified in the European club competitions – Steaua București, Astra Giurgiu and Pandurii Târgu Jiu, respectively relegated teams – SC Vaslui, Gaz Metan Mediaș, Viitorul Constanța and Corona Brașov, the “verdicts” provided by the comparative ranking system are truly and significantly different.

Obviously, an analysis conducted at the level of a single competition and for a single season cannot provide conclusive results based on which to support the idea that a pair-wise comparative ranking would produce a more objective hierarchy as opposed to that generated by the traditional ranking system. This is the first limit of the present research approach, fortunately one that can be surpassed by extending the number of the observation units and the period of observation.

A more important limit of this research approach refers to the fact that “reconstruction” of the 2013-2014 Romanian First Division final ranking in a comparative version has been done using the scores of the matches played under a different and “traditional” set of competition's rules. A reasonable doubt should be taken into consideration over how the matches would have been played and what scores registered if the competition would have been formatted based on the pair-wise comparison approach. Right now, it can only be supposed that a certain number of scores and, consequently, a variable number of the pair-games could have been ended differently...

Further points of discussion are represented by certain functional aspects of the pair-wise comparison approach. First, it could be more appealing to attribute two pair-wise points for the winner of a pair-game, a pair-wise point to each of the teams being in a perfectly symmetrical equality and zero pair-wise points to the loser of the pair-game. Thus, the maximum amount of pair-wise points to be obtained will be $2(n-1)$ (where “n” is the number of the participating teams) and will be avoided the remaining 0.5s resulted from splitting a single point.

Second, there could be taken into consideration a different way of ending a pair-game that

generated, after the home and away matches, a perfectly symmetrical tie. As in the recent history or the current organization of the European club competitions, playing additional extra-time periods (that may end when a team scores a “golden” goal, a “silver” goal or a “regular” goal that eliminate the tie), which may be followed by penalty shootouts, could solve the problem. Still, due to a rather psychological advantage created by the playing “at home” the “decisive” match (including the extra-time and/or the penalty shootout), host teams may have a moral benefit and the pair-game could be dis-balanced in their favor.

Under the circumstances described by the empirical conclusions, limits and elements of discussion generated by employment of the pair-wise comparison approach at the level of the 2013-2014 Romanian First Division, there are three concluding remarks that may be formulated.

First, the comparative ranking system may contribute to a more objective and sound hierarchy of a football league by expressing better the value of the teams. The champions should be better than as many as possible, ideally than all competing teams. Also, the relegated should be poorest by comparison to as many as possible, ideally by all of the competing teams.

Second, a competition of a higher intensity, in which each match counts for establishing the final hierarchy, is a realistic outcome created by the implementation of the pair-wise comparative ranking system. Teams with an increased motivation will actively dispute the championship, the places in the European club competitions and avoiding relegation.

Finally, more people attending more disputed matches may be another desirable outcome of the pair-wise comparative ranking system. With, practically, all teams motivated to play, home or away, to win the pair-games, get pair-wise points and ranking better in the final hierarchy, the competition will gain in attractiveness for all the stakeholders involved.

References

1. Baker, R. D., McHale, I. G., 2014. A dynamic paired comparisons model: Who is the greatest tennis player?, *European Journal of Operational Research*, 236, 677–684.
2. Boscá, J. E., Liern, V., Martínez, A., Sala, R., 2009. Increasing offensive or defensive efficiency? An analysis of Italian and Spanish football, *Omega The International Journal of Management Science*, 37, 63–78.
3. Brozos-Vázquez, M., Campo-Cabana, M. A., Díaz-Ramos, J. C., González-Díaz, J., 2008. Ranking participants in tournaments by means of rating functions, *Journal of Mathematical Economics*, 44, 1246–1256.
4. Da Silva, S., Matsushita, R., Silveira, E., 2013. Hidden power law patterns in the top European football leagues, *Physica A*, 392, 5376–5386.
5. Dahl, G., 2012. A matrix-based ranking method with application to tennis, *Linear Algebra and its Applications*, 437, 26–36.
6. Daniels, H. E., 1969. Round-robin tournament scores, *Biometrika*, 56, 295–299.
7. Glickman, M. E., Jensen, S. T., 2005. Adaptive paired comparison design, *Journal of Statistical Planning and Inference*, 127, 279–293.
8. Goddard, S.T., 1983. Ranking in tournaments and group decision making, *Management Science*, 29, 1384–1392.
9. Hvattum, L. M., Arntzen, H., 2010. Using ELO ratings for match result prediction in association football, *International Journal of Forecasting*, 26, 460–470.
10. Lebovic, J. H., Sigelman, L., 2001. The forecasting accuracy and determinants of football rankings, *International Journal of Forecasting*, 17, 105–120.
11. Moon, J. W., 1968. *Topics on Tournaments*, New York, Holt, Rinehart, and Winston.
12. Peterson, I., 1993. Who’s really #1? Statistical methods can influence college football ranking, *Science News*, 144, 412.

13. Rubinstein, A., 1980. Ranking the participants in a tournament, *SIAM Journal on Applied Mathematics*, 38, 108–111.
14. Sitarz, S., 2013. The medal points' incenter for rankings in sport, *Applied Mathematics Letters*, 26, 408–412.
15. Slutzki, G., Volij, O., 2005. Ranking participants in generalized tournaments, *International Journal of Game Theory*, 33, 255–270.
16. Stern, H. S., 1995. Who's number 1 in college football? . . . and how might we decide?, *Chance*, 8, 7–14.
17. Stob, M., 1985. Rankings from round robin tournaments, *Management Science*, 31, 1191–1195.
18. Stokkermans, K., 2014. Romania 2013/2014, the Rec.Sport.Soccer Statistics Foundation, available at <http://www.rsssf.com/tables/roem2014.html>, downloaded September 19, 2014.
19. Vegheș, C., 2014. S-ar fi putut termina și așa: Petrolul campioană, ACS Poli continuă în prima ligă!, *Fotbal Vest*, An 24, 1121, 14.